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POSSIBLE EXISTENCE OF PERIODIC TEMPERATURE FIELD DURING ELECTRICAL HEATING

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The temperature distribution in a metal rod heated by electric current is analyzed qualitatively.

Two other studies [1, 2] have dealt with the formation of dissipative three-dimensional structures in nonequilibrium systems. As examples of such structures were considered thermodynamically open mechanical, electrical, hydrodynamical, and chemical systems describable by nonlinear equations. A spatially organized steady structure under heavy thermodynamic fluxes can also form during heat transfer processes.

Just as was done in another study [3], we will here consider the steady temperature field in a metal rod through which there flows an electric current. The differential equation of the temperature field for the case of electrical resistivity $\rho = a + b\vartheta$, with a constant thermal conductivity λ and a heat transfer $q_s(\vartheta)$ which is some function of the temperature head, will be written as

$$\frac{d^2\vartheta}{dz^2} - \frac{\pi}{f\lambda} q_s(\vartheta) + \frac{bI^2}{f^2\lambda}\vartheta + \frac{aI^2}{f^2\lambda} = 0. \tag{1}$$

Equation (1) can be reduced to the autonomous system

$$\frac{d\vartheta}{dz} = y, \tag{2}$$

$$\frac{dy}{dz} = \frac{bI^2}{f^2\lambda}\left[\frac{f\pi}{bI^2} q_s(\vartheta) - \frac{a}{b} - \vartheta\right] = \frac{bI^2}{f^2\lambda} Q(\vartheta).$$

The simple state of equilibrium of system (2) is determined by the condition

$$y = 0, \quad Q(\vartheta) = \frac{f\pi}{bI^2} q_s(\vartheta) - \frac{a}{b} - \vartheta = 0, \quad \frac{dQ(\vartheta)}{d\vartheta} = 0. \tag{3}$$

It is evident from the expression for the roots of the characteristic equation of system (2)

$$k_{1,2} = \pm \frac{bI^2}{f^2\lambda}\left(\frac{dQ}{d\vartheta}\right)^{0,5}\Bigg|_{\vartheta=\vartheta_0} \tag{4}$$

that equilibrium states of both the saddle kind and the center kind can exist [4]. We have a saddle when

$$\frac{dQ}{d\vartheta}\Bigg|_{\vartheta=\vartheta_0} > 0, \tag{5}$$

and a center, i.e., a periodic $\vartheta(z)$ relation, when

$$\frac{dQ}{d\vartheta}\Bigg|_{\vartheta=\vartheta_0} > 0 \tag{6}$$

A complex state of equilibrium, with both roots of the characteristic equation equal to zero at points of standstill, cannot exist in system (2). Indeed, simultaneous vanishing of $Q(\vartheta)$ and $dQ/d\vartheta$ is equivalent to the differential equation

$$\frac{dq_s(\vartheta)}{d\vartheta} = \frac{b}{a + b\vartheta} q_s(\vartheta) \tag{7}$$

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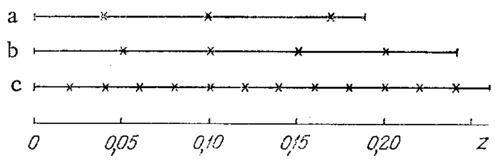

Fig. 1. Schematic distribution of glow zones along heated rods: (a) tungsten rod 4 mm in diameter, I = 4300 A, and $q_s = 1.3 \cdot 10^6$ W/m^2; (b) titanium tube 8 mm in diameter and 1.5 mm thick, I = 1350 A, and $q_s = 2.86 \cdot 10^6$ W/m^2; (c) steel (grade 0Kh18N10T) tube 8 mm in diameter and 1.5 mm thick, I = 1120 A, and $q_s = 2.89 \cdot 10^6$ W/m^2.

with a solution of the form

$$q_s(\vartheta) = \frac{bI^2}{f\pi}\left(\frac{a}{b} + \vartheta\right). \tag{8}$$

This relation contradicts the condition of zero thermal flux at $\vartheta = 0$ and thus has no physical meaning.

The existence of singular points and the character of the solution to Eq. (1) are determined by the form of function $q_s(\vartheta)$ and the parameters $f\pi/bI^2$, a/b but do not depend on the thermal conductivity of the metal. The bifurcation value of the parameter at which the topological structure of system (2) changes is

$$\frac{bI^2}{f\pi} = q_s(\vartheta)\Big|_{\vartheta=\vartheta_0}. \tag{9}$$

Expression (9), with the relation $q_s(\vartheta)$ known, yields the region where a periodic or monotonic temperature distribution along the heated rod can exist.

One usually finds that condition (5) is always satisfied in the case of a linear $q_s(\vartheta)$ relation, i.e., of a constant heat transfer coefficient, as well as in the case of heat transfer by radiation, inasmuch as then $q_s(\vartheta)/\vartheta > bI^2/f\pi$. In the case of pool boiling (the $q_s(\vartheta)$ curve is sufficiently well known for a wide range of heat loads) it is possible that condition (6) will be satisfied when a periodic solution exists. This possibility can be verified by direct calculation with the use of data on boiling of liquids at horizontal rod surfaces. Important is here the magnitude of the peak on the $q_s(\vartheta)$ curve, which depends strongly on the temperature rise needed to reach the boiling point [5].

In order to verify the existence of a periodic solution to Eq. (1), experiments were performed with water under atmospheric pressure at various temperatures under the boiling point. A typical distribution of temperature peaks along the test segment is shown in Fig. 1. Altogether, more than 100 tests were performed covering the range of q_s from $2 \cdot 10^6$ to $24 \cdot 10^6$ W/m^2. The diameter of the gauge segment was varied from 2 to 8 mm, specimens were made of tungsten, grade 0Kh18N10T stainless steel, and grade 7M titanium steel, respectively. The length of rods and thick-walled tubes varied over the 0.19-0.27 m range. A controllable rectifier served as source of power, delivering currents up to 6000 A at voltages of 0-18 V.

The periodicity of the $\vartheta(z)$ distribution was recorded visually, as an alternation of glow spots, i.e., of temperature peaks along the rod surface. The distance between glow spots varied from 0.015 m (grade 0Kh18N10T steel, 8 mm in diameter) to 0.07 m (tungsten, 4 mm in diameter).

The appearance of three dark spots above the experimental tube on the photograph in study [5] can be explained as manifestation of a periodic temperature field built up during the experiment with film boiling.

According to the results, local overheating of a heated surface under heavy heat loads is determined not only by restructurization of the hydrodynamic field in the boundary layer of fluid but also by heat generation and leakage within the heater body. This must be taken into consideration when data on heat transfer obtained in experiments with direct electrical heating are extended to other modes of heating (microwave, steam, nuclear, etc.).

NOTATION

I, electric current; l, f, π, respectively, the length, the cross-sectional area, and the perimeter of a metal rod; z, longitudinal coordinate; λ, thermal conductivity; ρ, electrical resistivity; a, b, coefficients in the temperature dependence of electrical resistivity; q_s, specific thermal flux; ϑ, temperature drop; and k, wave number.

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APPROXIMATE SOLUTION OF THE PROBLEM OF SOLIDIFICATION OF CURVILINEAR WALLS AND HOLLOW BODIES UNDER TRANSIENT CONDITIONS

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An approximate solution is obtained for the problem of solidification of curvilinear walls, hollow and continuous bodies in a transient process under conditions of operation of a set of thermal regime factors.

The operation of a large number of different instrumentation objects is characterized by a pulse heat liberation law in combination with high densities of the thermal fluxes being dissipated. This circumstance governed the broad application of melting materials and heat accumulators, in problems to support the thermal regimes of apparatus [1]. Unfortunately, the solution of nonstationary heat conduction problems associated with the phase transition process evokes substantial difficulties of a calculational nature. For this reason, the majority of known analytic solutions of the solidification problems includes bodies of canonical form (plates, cylinders, spheres) [2]. Of considerably greater interest in engineering is the possibility of analyzing the dynamics of hollow bodies and curvilinear shells, as well as of bodies of complex configuration.

Let us consider the problem of shell solidification (Fig. 1) characterized by the governing dimensions R_1 and R_2. Integral geometric properties of the shell are given by values of the inner S_1 and outer S_2 surfaces and the corresponding volumes V_1 and V_2.

The solidification process is determined by the action of two groups of regime factors:

1) actions from the surface S_1 of a medium with temperature t_1 and a heat source of density q_1 distributed uniformly over this surface;

2) actions from the surface S_2 of a medium with temperature t_2 and a surface heat source of density q_2.

The intensity of the heat exchange process on surfaces S_1 and S_2 is given by the heat transfer coefficients α_1 and α_2. The thermophysical characteristics (heat conduction coefficients λ_1 and λ_2, densities γ_1 and γ_2, heat of phase transition ρ) of the solid 1 and liquid 2 phases are considered given.

At the initial instant $\tau = 0$ the temperature of the medium t_2 is reduced to a temperature less than the solidification temperature t_3, and later remains constant.

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